Int. J. Heat Mass Transfer. Vol. 28, No. 2, pp. 361–369, 1985
Printed in Great Britain

0017-9310/85 \$3.00 + 0.00
Pergamon Press Ltd.

Unsteady convective heat transfer for turbulent flows of gases and liquids in tubes

E. K. KALININ
Research Corporation of the Ministry of Higher and Secondary Specialized Education of the RSFSR, Moscow, U.S.S.R.

and

G. A. DREITSER
Moscow Aviation Institute, Moscow, U.S.S.R.

(*Received* 19 *September* 1983)

Abstract—The paper presents an analysis and generalization of experimental data on the unsteady heat transfer of gases and liquids flowing in tubes under the conditions of heating/cooling and variation in the flow rate, heat release in the tube walls, and entrance flow temperature. The comparison between the results of theoretical analysis and experiments has made it possible to find the distribution of turbulent flow parameters over the tube radius and to determine the effect of variable flow structure on heat transfer. Different findings concerning the non-stationarity effects on turbulent flow characteristics are compared with the data of experiments. As a result, the correlations have been derived to calculate the unsteady heat transfer coefficient. Engineering calculation methods for real unsteady thermal processes are suggested.

1. INTRODUCTION

A CONSIDERABLE upsurge of interest shown at present in the problems of unsteady convective heat transfer in channels is motivated by that great part which the unsteady thermal processes play in modern power engineering equipment, heat exchange apparatus, and technological instruments, and also by more stringent requirements as to the accuracy of calculation of these highly rated devices. The unsteady thermal processes in these devices are characterized by high rates of parameter variations and, in a number of cases, are the determining ones. The calculations of unsteady thermal processes in the above-mentioned apparatus should be based on the results of fundamental investigations into non-stationary convective heat transfer processes. These investigations are needed to develop the reliable methods for calculation of temperature fields and thermal stresses, processes of heating and cooling of pipelines, manifolds, elements of power plants, to devise the methods for the optimization of these processes, calculation of transient, starting, and emergency regimes in various heat exchanging equipment employed in different branches of technology, for the development of automatic control systems.

In the majority of problems met in practice, the flow in channels is a turbulent one, therefore the unsteady turbulent heat transfer is considered in this work. The survey of non-stationary heat transfer studies is presented in references [1–3].

2. TECHNIQUE OF EXPERIMENTAL UNSTEADY HEAT TRANSFER DATA GENERALIZATION

The available theoretical investigations are based on the hypothesis of a quasistationary turbulent structure of flow, although the turbulent flow structure in unsteady conditions may differ substantially from the quasistationary one. For this reason, it is advisable that the theoretical methods be developed in conjunction with experimental studies. It should be noted that in spite of the availability of engineering recommendations, which can be found, e.g. in reference [1], up to now the practical calculations have often been based on the so-called quasistationary relations, when for each instant of the non-stationary process the heat transfer is calculated by the formulae of a stationary process with the parameters equal in magnitude to the instantaneous values of the non-stationary process parameters.

Generally, the unsteady heat transfer of a coolant flow in a tube is calculated in order to determine the non-stationary velocity and temperature fields in the coolant flow, the fields of temperatures and thermal stresses in the material of constructions surrounding the flow. In the majority of cases all that is necessary to know for the flow are only the mean mass temperatures, mean flow speed and pressure drops.

The problem of unsteady heat transfer in tubes is a conjugate one, since the mathematical model for the description of heat transfer and hydrodynamics of a coolant is augmented with the equations of heat conduction in the material of construction and with the conjugation conditions at the boundary between the coolant and the wall. As yet, the theoretical solution of this problem involves insurmountable difficulties posed by the impossibility of obtaining a closed system of equations for turbulent non-stationary flows because of the lack of experimental data on the turbulent flow structure in non-stationary and non-isothermal conditions.

NOMENCLATURE

a	thermal diffusivity
$a_0, \ldots, a_3$	coefficients
C, C_1, C_2	constants
c_p	heat capacity
d	internal diameter of tube
G	mass flow rate
g	acceleration due to gravity
f	function
i	enthalpy
K	non-stationary to quasistationary heat transfer, coefficient ratio, Nu/Nu_0
$K_q, K_{qx}, K^*_{Tg}, K_{Tg}, K^{**}_{Tg}$	thermal instability parameters
K_G	hydrodynamic instability parameter
ΔK_1	change of K due to non-stationary heat conduction
ΔK_2	change of K due to the difference between the turbulent flow structure in non-stationary and stationary conditions
ΔK_3	change of K under the influence of variable flow rate
l	tube length
Nu	Nusselt number
Pr	Prandtl number
q	heat flux density
r	radius
r_0	tube radius
R	$= r/r_0$
Re	Reynolds number
T	temperature
T_b	mean mass flow temperature
T_w	temperature of internal tube surface
T_w/T_b	temperature factor
w_x	axial gas velocity
x	axial coordinate.

Greek symbols

α	heat transfer coefficient
β	coefficient of volume expansion
ε_q	turbulent thermal diffusivity
η	dimensionless distance from the wall
λ	thermal conductivity
λ_T	turbulent thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
ξ	friction factor
ρ	density
τ	time.

Subscripts

0	quasistationary value
b	value determined by mean mass flow temperature
w	value determined by temperature of the internal tube surface.

In view of the fact that it is so difficult to construct the calculation methods based on the solution of three-dimensional conjugate problems, it seems to be most advisable then that these methods be based on the solution of conjugate problems with the processes in a coolant taken to be one-dimensional. Such an approach substantially simplifies the mathematical formulation of the problem making it quite solvable for numerical calculations on modern electronic computers. In this case, the heat conduction equation for the channel walls is augmented with one-dimensional equations of motion, energy and discontinuity of the flow. This system becomes closed if the relations for the heat transfer coefficient and friction factor are available. As a rule, these relations can be obtained only experimentally.

The heat transfer coefficients are

$$\alpha(x, \tau) = q_w(x, \tau)/[T_w(x, \tau) - T_b(x, \tau)]. \qquad (1)$$

In non-stationary conditions, the heat transfer is controlled not only by the parameters which characterize the stationary heat transfer (Reynolds and Prandtl numbers, distance from the entrance x/d, parameters allowing for the variability of coolant properties), but generally also by the laws which govern the variation of boundary conditions: flow rate G, wall temperature T_w or heat flux density on the wall q_w. In the case of turbulent flow, for the overwhelming majority of practically realizable laws, which govern the change in these conditions, it is possible to take only the linear expansion terms and to allow for the non-stationarity effect on heat transfer by the first derivatives of T_w or q_w with respect to time and length and of the flow rate with respect to time, or to take only the corresponding dimensionless parameters.

For a general non-stationary case of a turbulent flow in a channel, the relation for the Nusselt number has the form

$$Nu_b = f_1\left(\frac{x}{d}, Re_b, Pr_b, \frac{\mu_w}{\mu_b}, \frac{\lambda_w}{\lambda_b}, \frac{\rho_w}{\rho_b}, \frac{c_{pw}}{c_{pb}}, K_q, K_{qx}, K^*_{Tg}, K_G\right) \qquad (2)$$

where

$$K_q = \frac{\partial q_w}{\partial \tau}\frac{d^2}{q_w a}$$

takes into account the effect of $q_w(\tau)$ on convective heat transfer due to the superimposed non-stationary heat conduction; $K_{qx} = (\partial q_w/\partial x)(d/q_w)$ allows for the effect of q_w variation along the channel length (analogous relations can be obtained in terms of $\partial T_w/\partial\tau$ and $\partial T_w/\partial x$); the parameter $K_{Tg}^* = (\partial T_w/\partial\tau)\beta_w d\sqrt{(\lambda/c_p g G)}$ accounts for the effect of change in the turbulent flow structure on non-stationary heat transfer at variable T_w and constant flow rate; the parameter $K_G = (dG/d\tau)(d^2/G\nu)$ allows for the effect of flow rate variation.

Numerical calculations [4], made for the stationary heating of air at $Re_b = (1.6\text{–}2.3)\times 10^5$, $T_w/T_b = 1\text{–}2.2$, $K_{qx} = -0.01\text{–}0.013$ with allowance for the variable properties, and the experiment [5], carried out in an electrically heated tube with a variable wall thickness, $d = 6.05$ mm, length $l_1 = 1081$ mm at $Re_b = (2.24\text{–}17.3)\times 10^4$, $T_w/T_b = 1.06\text{–}2.2$ $|K_{qx}| = 0.005\text{–}0.013$, have shown that at the real values of the parameter K_{qx} its influence on heat transfer is inappreciable, and therefore was disregarded in data generalization.

The change of T_w in time exerts effect on the heat transfer rate due to the temperature profile rearrangement as a result of the non-stationary heat conduction superposition on the stationary convective heat transfer. When $\partial T_w/\partial\tau > 0$, the heat transfer is higher than in the stationary case ($K = Nu/Nu_0 > 1$) and at $\partial T_w/\partial\tau < 0$, it is lower ($K < 1$). The calculation of this effect for the turbulent air flow over the hydrodynamic stabilization length was made presuming the quasi-stationary structure of turbulence and allowing for the variability of gas properties. The gas flow rate was assumed constant, the quantity $q_w(x, \tau)$ increased with time.

The energy equation

$$\rho\frac{\partial i}{\partial\tau} + \rho w_x\frac{\partial i}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\lambda+\rho c_p\varepsilon_q)\frac{\partial T}{\partial r}r\right] \quad (3)$$

was solved numerically employing the radial heat flux density distribution approximation by the polynomial

$$q = (\lambda+\rho c_p\varepsilon_q)\frac{\partial T}{\partial r} = q_w(a_0+a_1R+a_2R^2+a_3R^3) \quad (4)$$

confirmed by preliminary calculations.

The turbulent flow structure was calculated by Richardt's formula [6]; for the variable properties to be taken into account, the dimensionless distance from the wall $\eta = (1-R)\cdot Re_w\sqrt{(\xi/32)}$ was determined from the values of ρ and μ at T_w. The calculation provided the convergence of the mean mass enthalpy found by integration, with the enthalpy value obtained from the solution of one-dimensional energy equation. Because of the high gas thermal diffusivity, the effect of non-stationary heat conduction was shown to be insignificant and much less than that measured experimentally (Fig. 1). Similar results were obtained by the finite-difference numerical solution of the problem performed on a BESM-6 computer for $Re_b = 10^4\text{–}3\times 10^4$. For liquids, this effect is more substantial

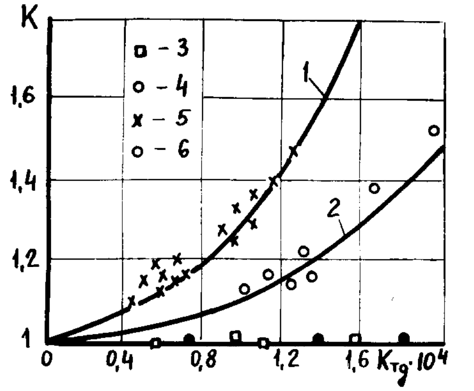

FIG. 1. Non-stationary heat transfer of a heated gas in a tube ($T_w/T_b = 1.1$):

$$K_{Tg} = \frac{\partial T_w}{\partial\tau}\frac{d}{(T_w-T_b)_0}\sqrt{\left(\frac{\lambda}{c_p g G}\right)}$$

$(T_w-T_b)_0$ is the final temperature difference; (1), (2) experimental data; (3), (4) calculation based on quasi-stationary turbulence; (5), (6) calculation based on non-stationary turbulence; (1), (3), (5) $Re_b = 10^5$; (2), (4), (6) $Re_b = 4.4\times 10^5$.

because of much lower thermal diffusivity: however, the experimental data also differ from the calculated results.

This difference is attributed to a change in the turbulent structure on heating or cooling of the wall layer. The turbulent thermal conductivity λ_T increases with heating of the wall layer and decreases with its cooling.

For the analysis of this phenomenon, the results of works [7, 8] were used, as well as the results of other works where it had been shown that in the wall layer, in the zone with $5 \leqslant \eta \leqslant 15$, there was a periodic origination of vortex structures which were ejected into more distant layers. It is the interaction of these ejections with the main stream, mainly in the zone $7 \leqslant \eta \leqslant 30$, which generates the turbulence, with the intensity and mean frequency of these ejections being the functions of the averaged-value parameters.

It may be presumed that under the conditions of non-stationary heating of the flow at $\partial T_w/\partial\tau > 0$, the retarded mass of gas near the wall has time to be heated substantially and to expand, thus increasing the surface of its interaction with large accelerated masses of a relatively cold gas and leading to a more intensive generation of turbulence. In the postulated mechanism of turbulence generation, the effect of non-stationary heating of the wall will be the stronger, the higher is the coefficient of the volume expansion of the gas near the wall, β_w, and the greater is the local change of flow temperature $\Delta T^* = \partial T_w/\partial\tau\,\Delta\tau^*$ in mean time between successive inceptions of vortex structures $\Delta\tau^*$, or the larger are the corresponding dimensionless parameters K_{Tg}^* or $K_{Tg}^{**} = (\partial T_w/\partial\tau)\beta_w\sqrt{(d/g)} = K_{Tg}^*\sqrt{(\pi\cdot Re\cdot Pr/4)}$.

For gases at the wall temperature, $\beta_w = 1/T_w$. The determination of the quantities $\rho, c_p, \lambda, a, \nu$, entering into

K_q, K^*_{Tg}, K^{**}_{Tg}, K_G, is based on T_b. As the experiments have shown, the effect of x/d on the non-stationary heat transfer is the same as on the quasistationary Nu_{b0}. The variability of liquid properties exerts an identical influence on Nu_b and Nu_{b0}, therefore one has

$$K = \frac{Nu_b}{Nu_{b0}} = f(Re_b, Pr_b, K_q, K^*_{Tg}, K_G) \quad (5)$$

or, with the separate allowance for the effect of the parameters K_q, K^*_{Tg}, K_G on unsteady heat transfer,

$$K = 1 + \Delta K_1(K_q, Re_b, Pr_b) + \Delta K_2(K^*_{Tg}, Re_b) + \Delta K_3(K_G, Re_b). \quad (6)$$

For gases Pr = const., while the effect of the variability of properties, taken into account by T_w/T_b, is different for Nu_b and Nu_{b0}, therefore

$$K = f(Re_b, T_w/T_b, K_q, K^*_{Tg}, K_G) = 1 + \Delta K_1(K_q, Re_b) + \Delta K_2(Re_b, T_w/T_b, K^*_{Tg}) + \Delta K_3(Re_b, T_w/T_b, K_G). \quad (7)$$

3. SHORT DESCRIPTION OF EXPERIMENTS AND FACILITIES

The experiments on unsteady heat transfer of gas and liquid flows in tubes were carried out under the following non-stationary boundary conditions:

(1) With constant flow rate and heating of gas in a tube, the wall temperature varied due to a stepwise or smooth variation of heat transfer from the tube wall.
(2) The tube wall was heated due to a sharp or smooth increase of the gas temperature at the tube entrance; the gas flow rate was kept constant.
(3) The flow rate of the gas heated in a tube varied and simultaneously heat release in the tube wall changed in such a way as to keep the wall temperature constant.
(4) The gas being heated and cooled, its flow rate varied and so did the wall temperature.
(5) The experiments under the conditions similar to 1 and 4, but with liquid heated in tubes.

The experiments were conducted in tubes with diameters $d = 5.39$–42.9 mm in the following ranges of parameters. For the gas: $Re_b = 6\times10^3$–6×10^5; the temperature factor $T_w/T_b = 0.3$–1.7; the rate of wall temperature change $\partial T_w/\partial\tau = -500$–$700$ K s^{-1} and the rate of flow discharge change $dG/d\tau = -0.024$–0.007 kg s^{-2}. For liquid: $Re_b = 5\times10^3$–10^5; $Pr_b = 2$–12; $Pr_b/Pr_w = 1$–3.7; $\partial T_w/\partial\tau = -120$–$318$ K s^{-1}; $dG/d\tau = -0.4$–0.5 kg s^{-2}.

In the investigated ranges of parameters, the ratio of the non-stationary heat transfer coefficient to its respective quasistationary value, $K = Nu/Nu_0$, changed within 0.4–3.5.

4. GENERALIZATION OF EXPERIMENTAL RESULTS

For practical calculations of the non-stationary heat transfer of gas and liquid flows in tubes with time variable heat release in the channel walls and coolant flow rate, the empirical relations have been obtained in references [2, 9].

The correlation for ΔK_1 with a differently varying q_w has been derived from the calculated results [10] in the form

$$\Delta K_1 = 26.6(K_q)^{0.71}/Re_b \cdot Pr_b^{0.6} \quad (8)$$

for $Re_b = 10^4$–10^6; $Pr_b = 1$–10; $K_q = 0$–4000; $x/d = 3.16$–197;

$$\Delta K_1 = 1/[1 - 2.4K_q/Re_b \cdot Pr_b^{0.6}] - 1 \quad (9)$$

for $K_q = -2000$–0; $Re_b = 10^4$–10^5; $Pr_b = 1$–10.

At $K_q > 0$, $\Delta K_1 > 0$; at $K_q < 0$, $\Delta K_1 < 0$.

The empirical formulae for ΔK_2 and ΔK_3 in the case of gas heating for differently varying T_w and G are as follows.

(1) In the case of wall temperature growth:

$$\Delta K_2 = (2 - 0.83T_w/T_b)(10.4 - 19.2\,Re_b\times10^{-5}) \times (K^*_{Tg}\times10^4)^{1.836 - 0.664Re_b\times10^{-5}} \quad (10)$$

for $K^*_{Tg} = 0$–0.4×10^4; $Re_b = 7\times10^3$–2.5×10^4; $T_w/T_b = 1$–1.7;

$$\Delta K_2 = (2 - 0.83T_w/T_b)(4.6 - 1.46\,Re_b\times10^{-5}) \times (K^*_{Tg}\times10^4)^{1.605 - 0.1Re_b\times10^{-5}} \quad (11)$$

for $K^*_{Tg} = 0$–0.4×10^{-4}; $Re_b = 2.5\times10^4$–2×10^5; $T_w/T_b = 1$–1.7;

$$\Delta K_2 = \frac{e^{[(1.45 - 7Re_b\times10^{-7})K^*_{Tg}\times10^4] - 1}}{e^{[0.4(2 - K^*_{Tg}\times10^4)Re_b\times10^{-5}]}} \quad (12)$$

for $K^*_{Tg} = 0$–1.1×10^{-4}; $Re_b = 8\times10^4$–4.5×10^5; $T_w/T_b = 1$–1.4.

(2) In the case of wall temperature decrease:

$$\Delta K_2 = -1.25(2 - T_w/T_b) \times [1 - (0.325 + 0.206\cdot Re_b\times10^{-5})] \times |K^*_{Tg}\times10^4|^{0.105Re_b\times10^{-5} - 0.27} \quad (13)$$

for $K^*_{Tg} = -0.4\times10^{-4}$–$0.05\times10^{-4}$; $Re_b = 7\times10^3$–2×10^5; $T_w/T_b = 1$–1.7;

$$\Delta K_2 = 1.25(2 - T_w/T_b) \times (4.85 - 2.2Re_b\times10^{-5})K^*_{Tg}\times10^4 \quad (14)$$

for $K^*_{Tg} = -0.05\times10^{-4}$–$0$; $Re_b = 7\times10^3$–2×10^5; $T_w/T_b = 1$–1.6;

$$\Delta K_2 = -(0.5T_w/T_b - 0.42)[1 - e^{(4.6\times10^4K^*_{Tg})}] \quad (15)$$

for $K^*_{Tg} = -10^{-4}$–0; $Re_b = 8\times10^4$–5.2×10^5; $T_w/T_b = 1$–1.6.

(3) In the case of flow rate increase:

$$\Delta K_3 = 0.004(4.1 - 1.9T_w/T_b)K_G^{2.4 - 1.4Re_b\times10^{-5}} \quad (16)$$

for $K_G = 0$–14; $Re_b = 10^4$–2.5×10^5; $T_w/T_b = 1$–1.7.

(4) In the case of flow rate decrease:

$$\Delta K_3 = [(0.915+0.08Re_b \times 10^{-5}) \times |K_G|^{0.25Re_b \times 10^{-5}-0.16}] \times (0.66+0.275T_w/T_b)-1 \quad (17)$$

for $K_G = -14$ to -0.01; $Re_b = 10^4$–2.5×10^5; $T_w/T_b = 1$–1.7.

For $K^*_{Tg} > 0$, the value of $\Delta K_2 > 0$; for $K^*_{Tg} < 0$, $\Delta K_2 < 0$ (Fig. 2). For $K_G > 0$, $\Delta K_3 > 0$, and for $K_G < 0$, $\Delta K_3 < 0$ (Fig. 3), with ΔK_3 varying the stronger, the smaller are Re_b and T_w/T_b.

The experiments carried out made it possible to assess the effect of variation of T_w on the turbulent flow structure. It was assumed that under the non-stationary conditions the stationary distribution of turbulent thermal diffusivity is retained, but the empirical factor B was introduced into the dimensionless distances from the wall η. When $K^*_{Tg} > 0$, $B > 0$. With an increase in T_w, the value of λ_T/λ (λ_T is the turbulent thermal conductivity) increased 2–4 times in the wall region with a moderate increase in K, and by 20–50% in the flow core (Fig. 4).

The relationships obtained for ΔK_1 and ΔK_2 have allowed the determination of correlation between these quantities in the case of variation in time of T_w and q_w for a gas heated in a tube. As is seen from equations (8), (10) and (11), when the heat load increases with $Re_b = \text{const.}$,

$$\Delta K_1 = C_1(K_q)^{0.71} \quad (18)$$

$$\Delta K_2 = C_2(K^*_{Tg})^n \quad (19)$$

where n lies within 1.796–1.405, when Re_b varies from 7×10^3 to 2×10^5.

Taking into account the relations for K_q and K^*_{Tg} and the expression $G = 0.25\pi d\mu_b Re_b$, one obtains, other

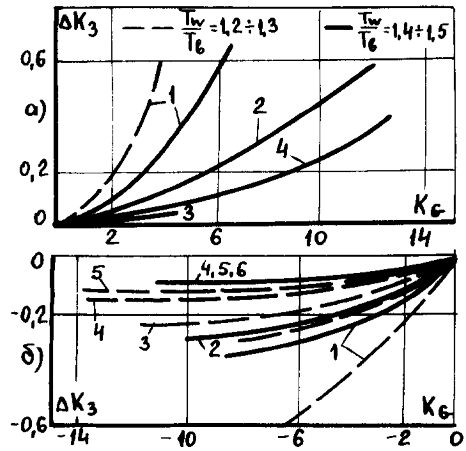

FIG. 3. ΔK_3 vs K_G at different Re_b and T_w/T_b for increasing (a) and decreasing (b) heated gas flow rate: (1)–(6), $Re_b\times10^4$ = 1–2; 2–3; 3–4; 4–5; 5–6; 6–7; (7) $T_w/T_b = 1.2$–1.3; (8) $T_w/T_b = 1.4$–1.5.

conditions being equal,

$$\Delta K_1/\Delta K_2 = Cd^m \quad (20)$$

where, for $Re_b = 7\times10^3$–2×10^5, $m = 0.522$–0.8075.

Thus, the ratio $\Delta K_1/\Delta K_2$ is the greater, the larger is the tube diameter. For $d = 10$–20 mm, the values which are usually employed in heat exchanging apparatus, $\Delta K_1 \ll \Delta K_2$ (Fig. 1), and it can be regarded that $\Delta K_1 = 0$, i.e. the difference of K from 1 is determined by the change in the turbulent flow structure. For large ds (e.g. when unsteady heat transfer is calculated for gas pipelines), ΔK_1 should be taken into account.

At constant flow rate and wall heating by a cooling gas, the empirical formula for ΔK_2 at differently varying

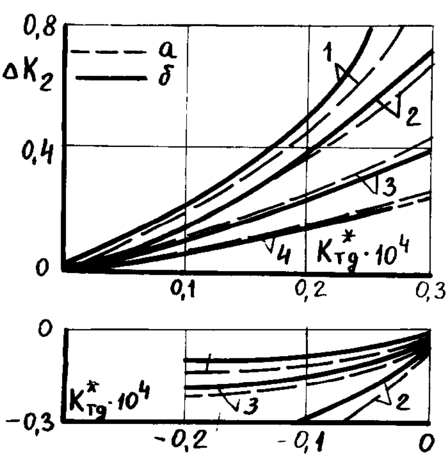

FIG. 2. Comparison of the dependence of ΔK_2 on K^*_{Tg} for sharp (a) and smooth (b) changes in the heat load in the case of gas heating: (1) $Re_b = (1$–$2)\times10^4$; $T_w/T_b = 1.1$–1.3; (2) $Re_b = (6$–$7)\times10^4$; $T_w/T_b = 1.1$–1.3; (3) $Re_b = (7$–$9)\times10^4$; $T_w/T_b = 1.3$–1.5; (4) $Re_b = (1.2$–$1.5)\times10^5$; $T_w/T_b = 1.5$–1.7; (5) $Re_b = (7$–$9)\times10^4$; $T_w/T_b = 1.5$–1.7.

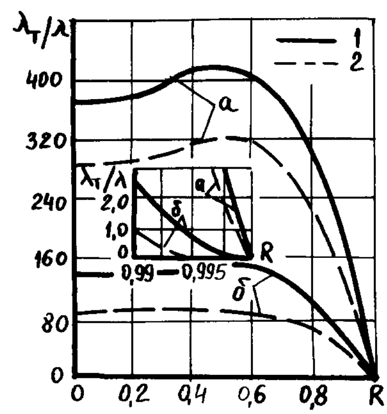

FIG. 4. Radial distribution of the turbulent thermal conductivity: (1) calculation with allowance for non-stationarity; (2) calculation in the quasistationary approximation: (a) $Re_b = 2.5\times10^5$; $T_w/T_b = 1.12$; $K_{Tg} = 1.26\times10^{-4}$; $K = 1.29$; (b) $Re_b = 5.5\times10^4$; $T_w/T_b = 1.16$; $K_{Tg} = 1.1\times10^{-4}$; $K = 1.45$.

wall temperature has the form

$$\Delta K_2 = \{[14.97(T_w/T_b)^3-16.07(T_w/T_b)^2 -0.526(T_w/T_b)+3.193] \times (Re_b \times 10^{-5})^{1.15-3T_w/T_b} +46.77(T_w/T_b)^3-119.1(T_w/T_b)^2 +99.09T_w/T_b-27.08\}K^*_{Tg} \times 10^5 \quad (21)$$

for $K^*_{Tg} = 0$–2×10^{-4}; $Re_b = 3.2 \times 10^4$–2×10^5; $T_w/T_b = 0.6$–1.

When the wall is heated up ($K^*_{Tg} > 0$), $\Delta K_2 > 0$. The value of ΔK_2 is the higher, the higher is K^*_{Tg} (Fig. 5). The non-stationarity effect increases with decreasing Re_b (Fig. 5) and T_w/T_b (Fig. 6).

The experiments with a periodic turn-on of hot gas discharge agree, despite different initial conditions of each cycle, with relation (21) and prove the validity of the latter for calculation of heat transfer at an arbitrarily varying T_w.

The empirical formulae for the case of liquid heating with differently varying T_w and G have the following form:

(1) With a change in the heat load, the value of $|\Delta K_2|$ is the greater, the greater is $|K^*_{Tg}|$ and the smaller is Re_b; it is independent of Pr_b and at $Pr_b = 3$–10 is correlated by the formulae

$$\Delta K_2 = (1.72 \times 10^6/Re_b^{0.303})K^*_{Tg} \quad (22)$$

for $Re_b = 4 \times 10^3$–2×10^4; $K^*_{Tg} = 0$–0.7×10^{-5};

$$\Delta K_2 = (8.29 \times 10^9/Re_b^{1.16})K^*_{Tg} \quad (23)$$

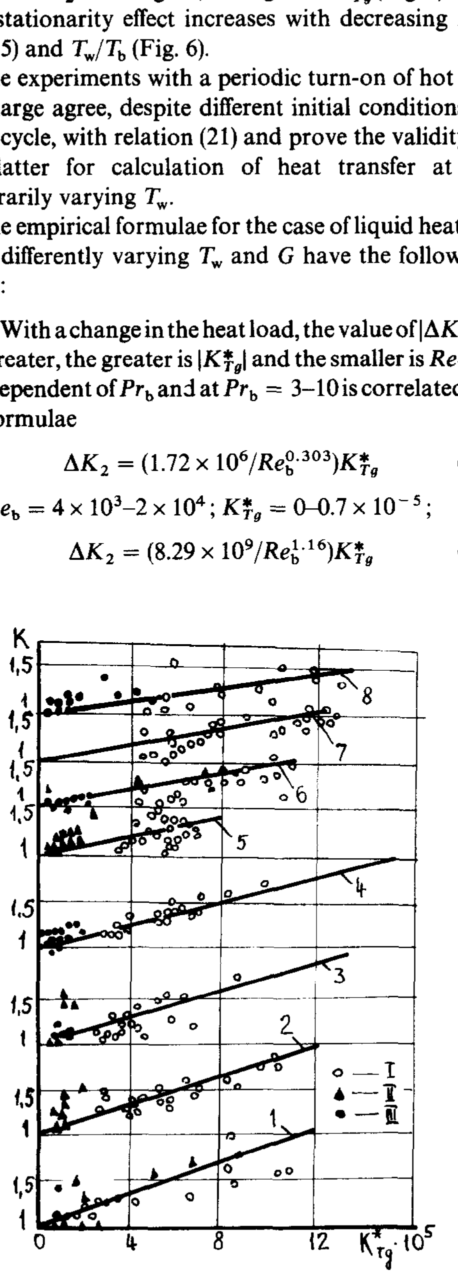

FIG. 5. K vs K^*_{Tg} in the case of gas cooling for $T_w/T_b = 0.7$–0.8 and different Re_b. (1)–(8) $Re_b \times 10^{-4} = 3.2$–$4$; 4–5; 5–6.25; 6.25–7.83; 7.83–9.83; 9.83–12.25; 12.25–15.5; 15.5–20. (I), (II), (III) tubes with $d = 8.65$; 42.8; 9.82 mm.

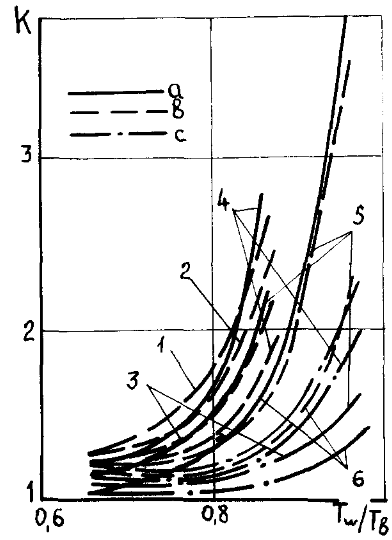

FIG. 6. K vs T_w/T_b for different Re_b and K^*_{Tg} in the case of gas cooling. (a) $Re_b = (3.2$–$4) \times 10^4$; (b) $Re_b = (6.25$–$7.83) \times 10^4$; (c) $Re_b = (1.55$–$2) \times 10^5$; (1)–(6), $K^*_{Tg} \times 10^5 = 6$; 5; 4; 3; 2; 1, respectively.

for $Re_b = 2 \times 10^4$–10^5; $K^*_{Tg} = 0$–0.7×10^{-5};

$$\Delta K_2 = (1-1.72 \times 10^6 K^*_{Tg}/Re_b^{0.303})^{-1}-1 \quad (24)$$

for $Re_b = 4 \times 10^3$–2×10^4; $K^*_{Tg} = -0.3 \times 10^{-5}-0$;

$$\Delta K_2 = (1-8.29 \times 10^9 K^*_{Tg}/Re_b^{1.16})^{-1}-1 \quad (25)$$

for $Re_b = 2 \times 10^4$–5×10^4; $K^*_{Tg} = -0.3 \times 10^{-5}$–$0$.

For liquids, the values of ΔK_1 and ΔK_2 are commensurable. The ratio $\Delta K_2/\Delta K_1$ decreases with an increase in the tube diameter at the same values of $\partial T_w/\partial \tau$, T_w, T_b, Re_b, Pr_b, therefore the dependence of K on K_q or K^*_{Tg} is not the same for tubes of different diameters, and it is necessary that two thermal instability parameters be used.

At the same values of K^{**}_{Tg} and Re_b, the values of ΔK_2 for a liquid and a gas (at T_w/T_b close to 1) actually coincide (Fig. 7), although the coefficients of volume expansion β_w for these differ by about 40 times. This confirms the validity of the proposed model of the effect of wall temperature change on the flow turbulence structure and non-stationary heat transfer, which is the stronger, the greater are $\partial T_w/\partial \tau$ and β_w.

The experiments carried out and their analysis have shown that the effect of turbulent flow structure change on non-stationary heat transfer is essential for both gases and liquids.

(2) The values of ΔK_3 have been found from the measured values of K and from the values of ΔK_1 and ΔK_2 obtained at $G =$ const. and at $Pr_b = 3$–12, $Pr_b/Pr_w = 1$–4; $x/d = 6$–160. These values are correlated by the following formulae (Fig. 8):

$$\Delta K_3 = (6 \times 10^{-9} K_G - 5.6 \times 10^{-6})Re_b - 7 \times 10^{-4} K_G - 0.071 \quad (26)$$

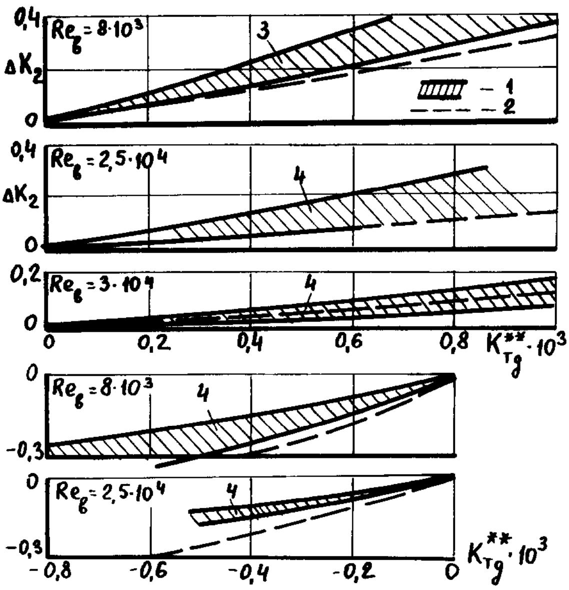

FIG. 7. Comparison of the data on the effect of change in the flow turbulence structure on non-stationary heat transfer in the case of gas and liquid heating: (1) water; (3) $Pr_b = 4-12$; (4) $Pr_b = 3$–12; (2) air; $Pr_b = 0.72$; $K^{**}_{Tg} = (\partial T_w/\partial\tau)\beta_w\sqrt{(d/g)}$.

for $Re_b = (6$–$12)\times 10^3$; $K_G = 0$–400;

$$\Delta K_3 = (9.3\times 10^{-8}K_G - 2\times 10^{-5})Re_b - 2.4\times 10^{-2}K_G + 0.236 \quad (27)$$

for $Re_b = (6$–$12)\times 10^3$; $K_G = -200$–0;

$$\Delta K_3 = (2.43\times 10^{-2}K_G - 5.67\times 10^{-2}) \times Re_b^{0.22} - (3.57\times 10^{-2}K_G - 0.83) \quad (28)$$

for $Re_b = (12$–$20)\times 10^3$; $K_G = -100$–200;

$$\Delta K_3 = (3.91\times 10^{-3}K_G + 2.173\times 10^{-6}) \times Re_b + 1.13\times 10^{-3}K_G^{-0.016} \quad (29)$$

for $Re_b = (20$–$60)\times 10^3$; $K_G = 0$–200;

$$\Delta K_3 = (-5\times 10^{-9}K_G - 2.75\times 10^{-6})Re_b + 2.8\times 10^{-3}K_G - 0.07 \quad (30)$$

for $Re_b = (20$–$60)\times 10^3$; $K_G = -100$–0.

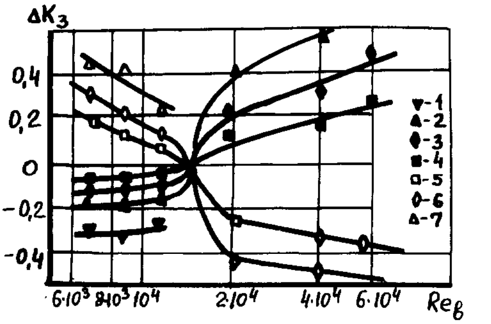

FIG. 8. ΔK_3 vs Re_b and K_G for an increasing ($K_G > 0$) and decreasing ($K_G < 0$) heated liquid flow rate: (1)–(7), $K_G = 400; 200; 100; 50; -50; -100; -200$.

When $Re_b = (1.5$–$6)\times 10^4$, $\Delta K_3 > 0$ at $K_G > 0$ and $\Delta K_3 < 0$ at $K_G < 0$.

With a decrease in Re_b, the effect of dynamic instability on heat transfer decreases and then reverses; with flow acceleration the heat transfer decreases and with flow deceleration it increases as compared with the quasistationary case.

Since, in real calculations, the values of T_w and $\partial T_w/\partial\tau$ (just as of q_w and $\partial q_w/\partial\tau$) are not known in advance, the problem is solved by the method of successive approximations. In the first approximation, the heat transfer coefficients are determined from the quasi-stationary relations. Then T_w and $\partial T_w/\partial\tau$, K^*_{Tg}, q_w, $\partial q_w/\partial\tau$, K_q and the non-stationary heat transfer coefficient are determined in the first approximation. This allows the next approximations to be made in the problem solution.

It should be noted that the empirical formulae presented in this section allow one, at the prescribed accuracy of calculation of the heat transfer coefficient, to determine the admissible rates of change in the parameters $\partial T_w/\partial\tau$, $\partial q_w/\partial\tau$, $dG/d\tau$ and the limits of applicability of the quasistationary correlations for the heat transfer coefficient [11]. For example, Fig. 9 presents the dependence on Re_b and T_w/T_b of the limiting values of the parameter K^*_{Tg} at which ΔK does not exceed the specified values for the case of hot gas cooling in tubes.

5. CONCLUSIONS

(1) In the case of a turbulent flow, the difference of the unsteady heat transfer coefficient from the quasi-stationary one is determined not by the laws which govern the variation of boundary conditions, but only

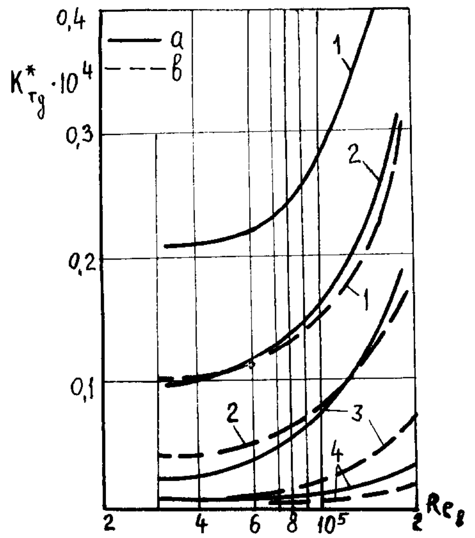

FIG. 9. The limiting values of K_{Tg}^* at which ΔK does not exceed 10% (a) and 5% (b); (1)–(4), T_w/T_b = 0.65; 0.75; 0.85; 0.95.

by the rates of their change, i.e. by the first derivatives of the flow rate, wall temperature or heat flux density on the wall. The corresponding parameters K_q, K_{Tg}^*, K_G have been obtained which determine the change in the heat transfer coefficient under non-stationary conditions.

(2) The experiments and their analysis have shown that the influence of a change in the flow turbulence structure on unsteady heat transfer is essential for both gases and liquids. Therefore, the calculations of unsteady heat transfer with the use of a quasistationary turbulence structure entail the errors impermissible for practice.

(3) The experimentally confirmed correlations have been derived to calculate the unsteady heat transfer coefficient for gas and liquid flows in tubes for the majority of practically encountered types of non-stationary effects over a wide range of parameters. In particular, these correlations make it possible, with the specified accuracy of calculations, to determine the range of validity of the quasistationary calculation method for unsteady thermal processes.

REFERENCES

1. V. K. Koshkin, E. K. Kalinin, G. A. Dreitser and S. A. Yarkho, *Unsteady Heat Transfer*. Izd. Mashinostroenie, Moscow (1973).
2. G. A. Dreitser and V. A. Kuzminov, *Calculation of Heating and Cooling of Pipelines*. Izd. Mashinostroenie, Moscow (1977).
3. N. M. Belyaev, A. A. Kochubei, A. A. Ryadno and V. F. Falii, *Nonstationary Heat Transfer in Tubes*. Izd. Vischa Shkola, Kiev (1980).
4. G. A. Dreitser and B. S. Baibikov, The effect of length-variable heat flux density on heat transfer for turbulent gas flow in tubes. In *Heat and Mass Exchange Between Flows and Surfaces*, Collected Papers of the Moscow Aviation Institute, No. 488, pp. 59–63 (1979).
5. G. A. Dreitser, An experimental investigation of heat transfer with the tube length-variable heat flux density. In *Heat and Mass Exchange Between Flows and Surfaces*, Collected Papers of the Moscow Aviation Institute, No. 351, pp. 64–68 (1976).
6. H. Richardt, Vollsfandige Darstellung der Turbulent Beschwindigkeits-Verteilung in Glatten Leitungen, *Z. angew. math. Mech.* **31**, 208–219 (1951).
7. E. R. Corino and R. S. Brodkey, Visual investigation of the wall region in a turbulent flow, *J. Fluid Mech.* **37**, 1–30 (1969).
8. S. J. Kline, W. S. Reynolds, F. A. Schraul and P. W. Renstadler, The structure of the turbulent boundary layers, *J. Fluid Mech.* **30**, 741–773 (1967).
9. G. A. Dreitser, P. M. Markovsky and V. D. Evdokimov, The influence of hydrodynamic instability on heat transfer of gas and liquid flows in tubes, *Izv. Akad. Nauk BSSR, Ser. Fiz.-Energ. Nauk* 3, 111–119 (1978).
10. Yu. N. Kuznetsov and V. P. Belousov, Numerical solution of the problem of nonstationary heat transfer for turbulent liquid flow in tubes, *Teplofiz. Vysok. Temp.* **8**, 148–1227 (1970).
11. G. A. Dreitser, Limits of applicability of quasistationary values of heat-transfer coefficients in calculating real nonstationary thermal processes, *J. Engng Phys.* **36**, 540–545 (1979).

CONVECTION THERMIQUE VARIABLE POUR DES GAZ ET DES LIQUIDES EN ECOULEMENT TURBULENT DANS DES TUBES

Résumé—On présente une analyse et une généralisation de résultats expérimentaux sur la convection thermique variable de gaz et de liquides en écoulement dans les tubes sous des conditions de chauffage et refroidissement et de variation des débits, de transfert thermique aux parois et de température d'entrée. La comparaison entre les résultats de l'analyse théorique et des expériences permet de trouver la distribution des paramètres de l'écoulement turbulent le long du rayon du tube et de déterminer l'effet de la structure variable de l'écoulement sur le transfert thermique. Différentes données concernant les effets de non stationnarité sur les caractéristiques de l'écoulement turbulent sont comparées avec l'expérience. Des formules sont données pour calculer le coefficient de convection variable. Des méthodes de calcul pratique sont suggérées pour les mécanismes thermiques réels variables.

INSTATIONÄRE KONVEKTIVE WÄRMEÜBERTRAGUNG BEI DER TURBULENTEN ROHRSTRÖMUNG VON GASEN UND FLÜSSIGKEITEN

Zusammenfassung—Versuchsdaten werden analysiert und in verallgemeinerter Form dargestellt, die bei der Messung des instationären Wärmeübergangs bei der Rohrströmung von Gasen und Flüssigkeiten unter folgenden Bedingungen aufgenommen worden sind: Heizen oder Kühlen, Veränderung des Durchflusses, der Wärmefreisetzung in der Rohrwand und der Eintrittstemperatur. Durch den Vergleich zwischen theoretischen und experimentellen Ergebnissen war es mögloch, die Verteilung der turbulenten Strömungsparameter längs des Rohrradius und den Einfluß veränderlicher Strömungsformen auf den Wärmeübergang zu bestimmen. Verschiedene Ergebnisse bezüglich der instationären Einflüsse auf die turbulente Strömung werden mit den experimentellen Daten verglichen. Korrelationen für die Berechnung des instationären Wärmeübergangskoeffizienten wurden entwickelt. Für reale instationäre thermische Vorgänge werden ingenieurmäßige Berechnungsmethoden vorgeschlagen.

НЕСТАЦИОНАРНЫЙ КОНВЕКТИВНЫЙ ТЕПЛООБМЕН ПРИ ТУРБУЛЕНТНОМ ТЕЧЕНИИ ГАЗОВ И ЖИДКОСТЕЙ В ТРУБАХ

Аннотация—Приводится анализ и обобщение экспериментальных данных по нестационарному теплообмену при течении газов и жидкостей в трубах в условиях нагревания и охлаждения при изменении расхода, тепловыделения в стенках каналов и температуры потока на входе. Сопоставление теоретического анализа с экспериментами позволило найти распределение турбулентных параметров потока по радиусу трубы и определить влияние переменной структуры потока на теплообмен. Приводится сопоставление различных данных по влиянию нестационарных воздействий на турбулентные характеристики потока с результатами экспериментов. В результате получены обобщающие зависимости для расчета нестационарного коэффициента теплоотдачи. Предложены инженерные методы расчета реальных нестационарных тепловых процессов.